36-710/36-752: Advanced Probability Overview Fall 2020

Instructor:

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TA:

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Lectures:

Monday and Wednesday, 8:00am - 9:20am, EST. Zoom link and recorded lectures available at this webpage (password protected).

Class Website:

http://www.stat.cmu.edu/~arinaldo/Teaching/36710-36752/F20/ Please check the website on a regular basis.

Prerequisites:

- Prerequisites: Calculus; Real analysis; Undergraduate Probability Theory.
- 36-705: Intermediate Statistics

As a general rule, if you already took a class on measure theory and integration (such as 21-720 or equivalent), you will not be allowed to register for this class.

Course Description

This is a one-semester course covering essential topics in probability underpinning theoretical statistics. One of the main goals of this course is to provide you with adequate theoretical background and mathematical tools to access the literature on mathematical statistics. The course is comprised of four modules: (1) basic measure theoretic concepts in probability; (2) modes of stochastic convergence and their applications in statistics; (3) martingales and stochastic processes and, if time allows, (4) a selection of modern topics in high-dimensional probability (such as concentration of measure, majorizing measure and Gaussian widths).

We will focus mostly on learning fundamental concets and basic proof techiques. We will only go over the proofs of selected material. If you are interested in a class on measure theory and advanced probability, you should look into taking such classes from the Math Department.

Please note that the structure and content of the course will be different than in past years. In particular, this is no longer a class in measure theory and probability.

Class material:

For most of modules 1 through 3 lecture notes will be provided. For the first module on measure theory and probability, the following books are recommended:

- Probability Theory & Measure Theory (1999), 2nd Ed., by R. Ash and C. Dolèans-Dale
- Probability, Theory and Examples (2019), 5th Ed., by R. Durrett.
- Probability and Measure (1995), 3rd edition, by P. Billingsley.
- A User's Guide to Measure Theoretic Probability (2002), by David Pollard.
- High-Dimensional Probability (2019) by R. Vershynin.

Further reading material and notes will be posted on the website.

Course Grading:

Your assessment and grades will be determined as follows:

- Homework assignments: 50%.
- Scribe duties: 5%.
- Attendance and class participation: 5%.
- Final Project: 40%.

Any failure to turn in any assignment, to fulfill the scribe duties and to miss a significant number of lectures without informing me of your absence or without a reasonable excuse will result in a lower grade.

Scribe duties:

Each student will take turn in transcribing the notes of every lecture in electronic format using the latex template available at the class Overleaf project. The public, view-only version of the Overleaf project for this class is here. To scribe a lecture you will need to access the editable version of the Overleaf project, whose link can be found on the password protected class webpage. The scribe has to attend class, take good and accurate notes, check for mistakes and inconsistencies, write them up in latex, add references and expand the material if appropriate and after consulting with me. The resulting pdf and latex files have to be submitted for the instructor's approval within one week. The pdf files containing the lecture notes will be posted publicly on the class website along with the names of the scribes. If you do not wish for your name to appear publicly for the lecture(s) you are scribing, please let the instructor know. The link to the scribe sign-up sheet is on the password protected class webpage.

Homework:

Homework problems will be assigned bi-weekly and are intended to give you an opportunity to check your understanding of the key concepts covered in class. The problems will be mostly of theoretical nature and may require you to prove/disprove statements.

There is a great value in discussing problems and sharing knowledge with your classmate, so you are encouraged to engage in group work. However, you should attempt to solve homework problems by yourself and only afterwards meet and compare with others.

Cheating or plagiarism on homework and exam will be dealt with to the full extent allowed under CMU policies: see https://www.cmu.edu/academic-integrity/defining/cheating.html

Attendance and Involvement:

It is important that you attend class, as the selection and organization of the topics will be on occasion different from the notes and textbooks. If you know you will be absent for few consecutive lectures, please let me know.

Come and see me any time you are confused or stuck and don't be shy in class: the more questions you ask and the more feedback I receive from you, the better I will be able to tailor the lectures to your specific needs.

Zoom rules of engagement:

The class will be taught remotely via Zoom and each lecture will be recorded. I will be sharing my screen and teaching out of my tablet. When the Zoom session starts your video will be on. Once the lecture starts, please make sure that you are muted while attending and feel free to turn off your video. If you want to ask a question during a lecture, just un-mute yourself (turn on your camera if it is off) and speak up. The class size is small enough that this type of interaction should be doable. Please do not post questions on the Zoom chat, as I will be focused my tablet and will not be monitoring the chat.

Final Project:

The final project involves picking a topic of interest, reading the relevant results in the area and then writing a short paper (8-12 pages) summarizing the key ideas in the area. You may focus on a single paper if you prefer. Your are NOT required to do new research, but you are welcome to. The paper should include background, statement of important results, and brief proof outlines for the results. If appropriate, you should also include numerical experiments are an application with real data.

- You may work by yourself or in teams of two.
- The goals are (i) to summarize key results in literature on a particular topic and (ii) present a summary of the theoretical analysis (results and proof sketch) of the methods (iii) implement some of the main methods. You may develop new theory if you like but it is not required.
- You will provide: (i) a proposal, (ii) a progress report and (iii) and final report.
- The reports should be well-written.

Proposal. <u>Due October 9.</u> A one page proposal. It should contain the following information: (1) project title, (2) team members, (3) precise description of the problem you are studying, (4) anticipated scope of the project, and (5) reading list. (Papers you will need to read).

Progress Report. Due <u>November 6.</u> Three pages. Include: (i) a high quality introduction, (ii) what have you done so far, (iii) what remains to be done and (iv) a clear description of the division of work among teammates, if applicable.

Final Report. Due <u>December 11.</u> The paper should be in NeurIPS format¹. (pdf only). Minimum 8 pages, maximum 12 pages. No appendix is allowed. You should submit a pdf file electronically. It should have roughly the following format:

¹The NuerIPS 201920style files can be found here: https://nips.cc/Conferences/2020/PaperInformation/StyleFiles

- 1. Introduction. Motivation and a quick summary of the area. Notation and Assumptions.
- 2. Key Results.
- 3. Proof outlines for the results.
- 4. Implementation (simulations or real data example.)
- 5. Conclusion. This includes comments on the meaning of the results and open questions.

Wellness:

Course work at this level can be intense, and I encourage you to take care of yourself. Do your best to maintain a healthy lifestyle by eating well, exercising, socializing, getting enough sleep and taking some time to relax. This will help you achieve your goals and cope with stress. All of us benefit from support during times of struggle. If you are having any problems or concerns, do not hesitate to come speak with me or the TA. There are also many resources available on campus that can provide help and support. Asking for support sooner rather than later is almost always a good idea. If you or anyone you know experiences any academic stress, difficult life events, or feelings like anxiety or depression, we strongly encourage you to seek support. Counseling and Psychological Services (CaPS) is here to help: call 412-268-2922 and visit their website at http://www.cmu.edu/counseling/. Consider also reaching out to a friend, faculty member, or family member you trust to help get you the support you need.

Diversity:

We must treat every individual with respect. We are diverse in many ways, and this diversity is fundamental to building and maintaining an equitable and inclusive campus community. Diversity can refer to multiple ways that we identify ourselves, including but not limited to race, color, national origin, language, sex, disability, age, sexual orientation, gender identity, religion, creed, ancestry, belief, veteran status, or genetic information. Each of these diverse identities, along with many others not mentioned here, shape the perspectives our students, faculty, and staff bring to our campus. We, at CMU, will work to promote diversity, equity and inclusion not only because diversity fuels excellence and innovation, but because we want to pursue justice. We acknowledge our imperfections while we also fully commit to the work, inside and outside of our classrooms, of building and sustaining a campus community that increasingly embraces these core values. Each of us is responsible for creating a safer, more inclusive environment. Unfortunately, incidents of bias or discrimination do occur, whether intentional or unintentional. They contribute to creating an unwelcoming environment for individuals and groups at the university. Therefore, the university encourages anyone who experiences or observes unfair or hostile treatment on the basis of identity to speak out for justice and support, within the moment of the incident or after the incident has passed. Anyone can share these experiences using the following resources:

- Center for Student Diversity and Inclusion: csdi@andrew.cmu.edu, (412) 268-2150
- Report-It online anonymous reporting platform: **reportit.net** username: *tartans* password: *plaid*

All reports will be documented and deliberated to determine if there should be any following actions. Regardless of incident type, the university will use all shared experiences to transform our campus climate to be more equitable and just.

Disability Resources

If you require a special accommodation, please visit the Office of Disability Resources to obtain appropriate documentation. See http://www.cmu.edu/hr/eos/disability/students/